

AI for special education: solving mathematical word problems with conceptual models approach using LLMs

Keywords: mathematical word problem, students with learning disabilities in mathematics, conceptual model-based problem solving, large language models (LLM), AI learning tool

Solving arithmetic word problems (WP) is an important, but also challenging, part of any school mathematics curriculum¹. Today's mathematical education standards emphasize *conceptual* understanding of mathematical relations in problem solving. For students with learning disabilities in mathematics (LDM), it is challenging to learn how to solve word problems in the relational paradigm, as it involves various skills, such as comprehension of the problem, generating a model equation for solution and performing required calculations. Mathematical education scholars proposed the conceptual model-based problem solving (MBPS) approach that has proven itself to be effective in enhancing WP-solving performance for students with LDM (Xin et al., 2023). The main idea of this approach is to represent a WP as a defined mathematical model expressed in an algebraic equation that drives the solution plan, along with a set of WP story grammar self-questioning prompts to facilitate conceptual understanding of mathematical relations in WPs². We propose to develop an AI tool that could automatically generate a correct step-by-step solution to WPs with additional capacity of enriching the text of arithmetic WPs with the conceptual model equations and ability of developing a set of story grammar questions. Large Language Models (LLMs) can generate step-by-step solutions to arithmetic word problems by analyzing the problem's language and structure. Trained on vast datasets, LLMs identify key components, form an algebraic equation based on the problem, and walk through each step of the solution. They can also provide guiding questions to reinforce understanding.

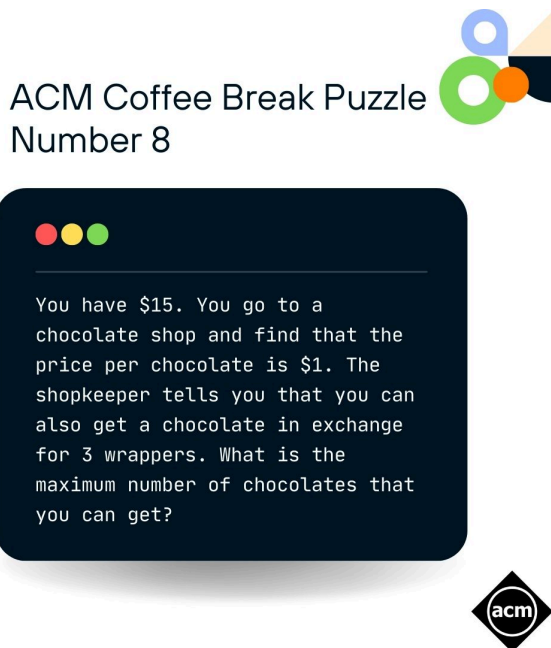
The project has the potential to provide personalized support for students and help educators.

¹ Word problems are math questions presented in the form of a narrative rather than equations or formulas alone. They describe a real-world situation using words and require the solver to translate the text into a mathematical problem, set up equations, and find a solution. For example: "Sarah has 3 apples and buys 5 more. How many apples does she have now?".

² For example, the conceptual model of the word problem "Alice has 2 apples, Bob has 3 apples. How many apples do they have together?" is a Part-Part-Whole (PPW), describing an additive relation between multiple parts and the whole, and comprising a set of variations. The above WP is a combine-PPW type, where the whole is unknown. Compare, with "Alice and Bob have found out that together they have 5 apples. Alice says that she has 3 apples. How many apples does Bob have?", which is the combine-PPW type of problem, where a part is unknown, and "Alice has 2 apples. Bob has her 3 more apples. How many apples does Alice have now?", which is a change-join-PPW problem, where the whole is unknown. Researchers consider different variations in additive and multiplicative problem structure, each with a set of problem variations (Xin, 2008), (Witzel, 2022). See the Appendix for the complete list.

Automatic solving of mathematical WPs is a challenging research problem in the fields of artificial intelligence (AI) since understanding and extracting relevant information from an unstructured text requires a lot of logical skills. Applying various AI based techniques in solving WPs is an attractive and quickly developing area (Davis, 2023). Recently, a rule-based approach was used for building the arithmetic WP solver, where the WPs were classified into four categories (Change, Combine, Compare, and Division-Multiplication) to identify the desired operation among the four main arithmetic operations. We are not aware, though, of existing tools that could automatically generate a correct step-by-step solution to WPs and, in addition, enrich the solution with a compact conceptual representation of the WP (Acharya et al., 2022).

Here is a math problem and how ChatGPT is solving it :



Let's solve this puzzle step by step.

1. **Initial Purchase:** You have \$15, and each chocolate costs \$1, so you can initially buy 15 chocolates.
2. **Wrappers Collected:** After eating 15 chocolates, you have 15 wrappers.
3. **Exchanging Wrappers:** You can exchange 3 wrappers for 1 chocolate.
 - From 15 wrappers, you can exchange them for 5 more chocolates (since $15 \div 3 = 5$).
4. **New Wrappers:** Now, you eat the 5 additional chocolates, giving you 5 new wrappers.
5. **Further Exchanges:** You can exchange the 5 wrappers for 1 more chocolate (since $5 \div 3 = 1$, with 2 wrappers left).

6. **Final Wrappers:** You eat this chocolate, gaining 1 wrapper, adding it to the 2 wrappers you already have, totaling 3 wrappers.

7. **Last Exchange:** Finally, you can exchange these 3 wrappers for 1 more chocolate.

Total Chocolates:

- 15 (initial) + 5 (first exchange) + 1 (second exchange) + 1 (final exchange) = 22 chocolates.

So, the maximum number of chocolates you can get is 22.

Literature:

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